5.5

Polynomials - Multiplying Polynomials

Objective: Multiply polynomials.

Multiplying polynomials can take several different forms based on what we are multiplying. We will first look at multiplying monomials, then monomials by polynomials and finish with polynomials by polynomials.

Multiplying monomials is done by multiplying the numbers or coefficients and then adding the exponents on like factors. This is shown in the next example.

Example 1.

$(4x^3y^4z)(2x^2y^6z^3)$	Multiply numbers and add exponents for x, y, and z
$8x^5y^{10}z^4$	Our Solution

In the previous example it is important to remember that the z has an exponent of 1 when no exponent is written. Thus for our answer the z has an exponent of $1+3=4$. Be very careful with exponents in polynomials. If we are adding or subtracting the exponnets will stay the same, but when we multiply (or divide) the exponents will be changing.

Next we consider multiplying a monomial by a polynomial. We have seen this operation before with distributing through parenthesis. Here we will see the exact same process.

Example 2.

$4x^3(5x^2-2x+5)$	Distribute the $4x^3$, multiplying numbers, adding exponents
$20x^5-8x^4+20x^3$	Our Solution

Following is another example with more variables. When distributing the exponents on a are added and the exponents on b are added.

Example 3.

$2a^3b(3\mathrm{ab}^2-4a)$	Distribute, multiplying numbers and adding exponents
$6a^4b^3-8a^4b$	Our Solution

There are several different methods for multiplying polynomials. All of which work, often students prefer the method they are first taught. Here three methods will be discussed. All three methods will be used to solve the same two multiplication problems.

Multiply by Distributing

Just as we distribute a monomial through parenthesis we can distribute an entire polynomial. As we do this we take each term of the second polynomial and put it in front of the first polynomial.

Example 4.

$$\begin{array}{rl} (4x+7y)(3x-2y) & \text{Distribute } (4x+7y) \text{ through parenthesis} \\ 3x\boldsymbol{(4x+7y)}-2y\boldsymbol{(4x+7y)} & \text{Distribute the } 3x \text{ and } -2y \\ 12x^2+21xy-8xy-14y^2 & \text{Combine like terms } 21xy-8xy \\ 12x^2+13xy-14y^2 & \text{Our Solution} \end{array}$$

This example illustrates an important point, the negative/subtraction sign stays with the $2y$. Which means on the second step the negative is also distributed through the last set of parenthesis.

Multiplying by distributing can easily be extended to problems with more terms. First distribute the front parenthesis onto each term, then distribute again!

Example 5.

$$\begin{array}{rl} (2x-5)(4x^2-7x+3) & \text{Distribute } (2x-5) \text{ through parenthesis} \\ 4x^2\boldsymbol{(2x-5)}-7x\boldsymbol{(2x-5)}+3\boldsymbol{(2x-5)} & \text{Distribute again through each parenthesis} \\ 8x^3-20x^2-14x^2+35x+6x-15 & \text{Combine like terms} \\ 8x^3-34x^2+41x-15 & \text{Our Solution} \end{array}$$

This process of multiplying by distributing can easily be reversed to do an important procedure known as factoring. Factoring will be addressed in a future lesson.

Multiply by FOIL

Another form of multiplying is known as FOIL. Using the FOIL method we multiply each term in the first binomial by each term in the second binomial. The letters of FOIL help us remember every combination. F stands for First, we multiply the first term of each binomial. O stand for Outside, we multiply the outside two terms. I stands for Inside, we multiply the inside two terms. L stands for Last, we multiply the last term of each binomial. This is shown in the next example:

Example 6.

$$\begin{array}{rl} (4x+7y)(3x-2y) & \text{Use FOIL to multiply} \\ (4x)(3x)=12x^2 & F-\text{First terms } (4x)(3x) \\ (4x)(-2y)=-8xy & O-\text{Outside terms } (4x)(-2y) \\ (7y)(3x)=21xy & I-\text{Inside terms } (7y)(3x) \\ (7y)(-2y)=-14y^2 & L-\text{Last terms } (7y)(-2y) \\ 12x^2-8xy+21xy-14y^2 & \text{Combine like terms } -8xy+21xy \\ 12x^2+13xy-14y^2 & \text{Our Solution} \end{array}$$

Some students like to think of the FOIL method as distributing the first term $4x$ through the $(3x-2y)$ and distributing the second term $7y$ through the $(3x-2y)$. Thinking about FOIL in this way makes it possible to extend this method to problems with more terms.

Example 7.

$$\begin{array}{rl}
(2x-5)(4x^2-7x+3) & \text{Distribute } 2x \text{ and } -5 \\
(2x)(4x^2)+(2x)(-7x)+(2x)(3)-5(4x^2)-5(-7x)-5(3) & \text{Multiply out each term} \\
8x^3-14x^2+6x-20x^2+35x-15 & \text{Combine like terms} \\
8x^3-34x^2+41x-15 & \text{Our Solution}
\end{array}$$

The second step of the FOIL method is often not written, for example, consider the previous example, a student will often go from the problem $(4x+7y)(3x-2y)$ and do the multiplication mentally to come up with $12x^2-8xy+21xy-14y^2$ and then combine like terms to come up with the final solution.

Multiplying in rows

A third method for multiplying polynomials looks very similar to multiplying numbers. Consider the problem:

$$\begin{array}{rl}
35 & \\
\underline{\times\, 27} & \\
245 & \text{Multiply 7 by 5 then 3} \\
\underline{700} & \text{Use 0 for placeholder, multiply 2 by 5 then 3} \\
945 & \text{Add to get Our Solution}
\end{array}$$

World View Note: The first known system that used place values comes from Chinese mathematics, dating back to 190 AD or earlier.

The same process can be done with polynomials. Multiply each term on the bottom with each term on the top.

Example 8.

$$\begin{array}{rl}
(4x+7y)(3x-2y) & \text{Rewrite as vertical problem} \\
4x+7y & \\
\underline{\times\, 3x-2y} & \\
-8xy-14y^2 & \text{Multiply } -2y \text{ by } 7y \text{ then } 4x \\
\underline{12x^2+21xy \qquad\quad} & \text{Multiply } 3x \text{ by } 7y \text{ then } 4x\text{. Line up like terms} \\
12x^2+13xy-14y^2 & \text{Add like terms to get Our Solution}
\end{array}$$

This same process is easily expanded to a problem with more terms.

Example 9.

$$\begin{array}{rl}
(2x-5)(4x^2-7x+3) & \text{Rewrite as vertical problem}\\
4x^3-7x+3 & \text{Put polynomial with most terms on top}\\
\underline{\times\, 2x-5} & \\
-20x^2+35x-15 & \text{Multiply } -5 \text{ by each term}\\
\underline{8x^3-14x^2+6x\qquad} & \text{Multiply } 2x \text{ by each term. Line up like terms}\\
8x^3-34x^2+41x-15 & \text{Add like terms to get our solution}
\end{array}$$

This method of multiplying in rows also works with multiplying a monomial by a polynomial!

Any of the three described methods work to multiply polynomials. It is suggested that you are very comfortable with at least one of these methods as you work through the practice problems. All three methods are shown side by side in the example.

Example 10.

$$(2x-y)(4x-5y)$$

Distribute	**FOIL**	**Rows**
$4x(2x-y)-5y(2x-y)$	$2x(4x)+2x(-5y)-y(4x)-y(-5y)$	$2x-y$
$8x^2-4xy-10xy-5y^2$	$8x^2-10xy-4xy+5y^2$	$\underline{\times\, 4x-5y}$
$8x^2-14xy-5y^2$	$8x^2-14xy+5y^2$	$-10xy+5y^2$
		$\underline{8x^2-4xy\qquad}$
		$8x^2-14xy+5y^2$

When we are multiplying a monomial by a polynomial by a polynomial we can solve by first multiplying the polynomials then distributing the coefficient last. This is shown in the last example.

Example 11.

$$\begin{array}{rl}
3(2x-4)(x+5) & \text{Multiply the binomials, we will use FOIL}\\
3(2x^2+10x-4x-20) & \text{Combine like terms}\\
3(2x^2+6x-20) & \text{Distribute the 3}\\
6x^2+18x-60 & \text{Our Solution}
\end{array}$$

A common error students do is distribute the three at the start into both parenthesis. While we can distribute the 3 into the $(2x-4)$ factor, distributing into both would be wrong. Be careful of this error. This is why it is suggested to multiply the binomials first, then distribute the coeffienct last.

5.5 Practice - Multiply Polynomials

Find each product.

1) $6(p-7)$

2) $4k(8k+4)$

3) $2(6x+3)$

4) $3n^2(6n+7)$

5) $5m^4(4m+4)$

6) $3(4r-7)$

7) $(4n+6)(8n+8)$

8) $(2x+1)(x-4)$

9) $(8b+3)(7b-5)$

10) $(r+8)(4r+8)$

11) $(4x+5)(2x+3)$

12) $(7n-6)(n+7)$

13) $(3v-4)(5v-2)$

14) $(6a+4)(a-8)$

15) $(6x-7)(4x+1)$

16) $(5x-6)(4x-1)$

17) $(5x+y)(6x-4y)$

18) $(2u+3v)(8u-7v)$

19) $(x+3y)(3x+4y)$

20) $(8u+6v)(5u-8v)$

21) $(7x+5y)(8x+3y)$

22) $(5a+8b)(a-3b)$

23) $(r-7)(6r^2-r+5)$

24) $(4x+8)(4x^2+3x+5)$

25) $(6n-4)(2n^2-2n+5)$

26) $(2b-3)(4b^2+4b+4)$

27) $(6x+3y)(6x^2-7xy+4y^2)$

28) $(3m-2n)(7m^2+6mn+4n^2)$

29) $(8n^2+4n+6)(6n^2-5n+6)$

30) $(2a^2+6a+3)(7a^2-6a+1)$

31) $(5k^2+3k+3)(3k^2+3k+6)$

32) $(7u^2+8uv-6v^2)(6u^2+4uv+3v^2)$

33) $3(3x-4)(2x+1)$

34) $5(x-4)(2x-3)$

35) $3(2x+1)(4x-5)$

36) $2(4x+1)(2x-6)$

37) $7(x-5)(x-2)$

38) $5(2x-1)(4x+1)$

39) $6(4x-1)(4x+1)$

40) $3(2x+3)(6x+9)$

Answers to Multiply Polynomials

1) $6p-42$

2) $32k^2+16k$

3) $12x+6$

4) $18n^3+21n^2$

5) $20m^5+20m^4$

6) $12r-21$

7) $32n^2+80n+48$

8) $2x^2-7x-4$

9) $56b^2-19b-15$

10) $4r^2+40r+64$

11) $8x^2+22x+15$

12) $7n^2+43n-42$

13) $15v^2-26v+8$

14) $6a^2-44a-32$

15) $24x^2-22x-7$

16) $20x^2-29x+6$

17) $30x^2-14xy-4y^2$

18) $16u^2+10uv-21v^2$

19) $3x^2+13xy+12y^2$

20) $40u^2-34uv-48v^2$

21) $56x^2+61xy+15y^2$

22) $5a^2-7ab-24b^2$

23) $6r^3-43r^2+12r-35$

24) $16x^3+44x^2+44x+40$

25) $12n^3-20n^2+38n-20$

26) $8b^3-4b^2-4b-12$

27) $36x^3-24x^2y+3xy^2+12y^3$

28) $21m^3+4m^2n-8n^3$

29) $48n^4-16n^3+64n^2-6n+36$

30) $14a^4+30a^3-13a^2-12a+3$

31) $15k^4+24k^3+48k^2+27k+18$

32) $42u^4+76u^3v+17u^2v^2-18v^4$

33) $18x^2-15x-12$

34) $10x^2-55x+60$

35) $24x^2-18x-15$

36) $16x^2-44x-12$

37) $7x^2-49x+70$

38) $40x^2-10x-5$

39) $96x^2-6$

40) $36x^2+108x+81$